Numerical Methods for Partial Differential Equations

Course Description: In these lectures we provide an introduction to the important classes of numerical methods for partial differential equations including finite difference, finite volume and Fourier-based spectral method. After fundamentals of numerical approximation are established, some current efficient approaches are presented. The emphasis is on a solid understanding of the accuracy of these methods. Computer implementations will be demonstrated in Matlab. The class is suitable for graduate students from all disciplines who have interest in computational PDEs. *Prerequisite: PDEs, Fourier Analysis, Linear Algebra*

Table of Contents

- Finite Difference Method
  - fundamentals of FD, consistency
  - 1D diffusion equation, boundary conditions
  - explicit and implicit methods for ODEs
  - stability: von Neumann analysis, Lax Equivalence Theorem
  - CFL condition, super time-stepping, iterative procedures
  - 2D and 3D problems, ADI methods
- Finite Volume Method
  - first order hyperbolic conservation laws
  - nonlinear advection, shock, Riemann problems
  - shock capturing schemes, Godunov, Limiters, TVD, ENO
- Spectral Method
  - interpolation, differentiation, FFT, Chebyshev grid, spectral accuracy
  - nonlinear waves

References:

1. LeVeque, R. J., *Finite Difference Methods for Ordinary and Partial Differential Equations*, SIAM, 2007.
2. LeVeque, R. J., *Finite Volume Methods for Hyperbolic Problems*, Cambridge University Press, 2002.
3. Boyd, J.P., *Chebyshev and Fourier Spectral Methods*, Dover, 2000.
4. Trefethen, L. N., *Spectral Methods in Matlab*, SIAM, 2000.