Solve Problem 1, then choose 3 others. Please put a box around all final answers! If attempting more than 4, write "Do Not Grade" on the problem that you do not want to have graded. (Sorry, there is no "best of" option.)

For each problem, show as much work as possible on the pages provided (and the backs of the pages if necessary). You may find the following tables useful.

$$\begin{aligned}
k_e &= \frac{1}{4\pi\epsilon_o} \simeq 9 \times 10^9 \quad [\mathrm{N \cdot m^2/C^2}] \\
\epsilon_o &= 8.854 \times 10^{-12} \quad [\mathrm{C^2/N \cdot m^2}] \text{ or } [\mathrm{F/m}] \\
\mu_o &= 4\pi \times 10^{-7} \quad [\mathrm{N/A^2}] \text{ or } [\mathrm{H/m}] \\
c &= 2.99792458 \times 10^8 \simeq 3 \times 10^8 \quad [\mathrm{m/s}] \\
m_e &= 9.109 \times 10^{-31} \quad [\mathrm{kg}] \\
m_p &= 1.673 \times 10^{-27} \quad [\mathrm{kg}] \\
q_e &= 1.602 \times 10^{-19} \quad [\mathrm{C}]
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{x} &= lnx \\
\int e^{ax}dx &= \frac{1}{a}e^{ax} \\
\int \frac{dx}{\sqrt{a^2 - x^2}} &= \arcsin\frac{x}{a} \\
\int \frac{dx}{\sqrt{x^2 + a^2}} &= \ln\left(x + \sqrt{x^2 + a^2}\right) \\
\int \frac{xdx}{\sqrt{x^2 + a^2}} &= \sqrt{x^2 + a^2} \\
\int \frac{dx}{x^2 + a^2} &= \frac{1}{a}\arctan\frac{x}{a} \\
\int \frac{dx}{(x^2 + a^2)^{3/2}} &= \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}} \\
\int \frac{xdx}{(x^2 + a^2)^{3/2}} &= -\frac{1}{\sqrt{x^2 + a^2}}
\end{aligned}$$

$$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \times \mathbf{A} = \mathbf{C} \cdot \mathbf{A} \times \mathbf{B}$$
$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$
$$\nabla(\psi V) = \psi \nabla V + V \nabla \psi$$
$$\nabla \cdot (\psi \mathbf{A}) = \psi \nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla \psi$$
$$\nabla \times (\psi \mathbf{A}) = \psi \nabla \times \mathbf{A} + \nabla \psi \times \mathbf{A}$$
$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$
$$\nabla \cdot \nabla V = \nabla^2 V$$
$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$
$$\nabla \times \nabla V = 0$$
$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$
$$\int_V \nabla \cdot \mathbf{A}\, dv = \oint_S \mathbf{A} \cdot d\mathbf{s} \qquad \text{(Divergence theorem)}$$
$$\int_S \nabla \times \mathbf{A} \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\boldsymbol{\ell} \qquad \text{(Stokes's theorem)}$$

Cartesian Coordinates (x, y, z)

$$\nabla V = \mathbf{a}_x \frac{\partial V}{\partial x} + \mathbf{a}_y \frac{\partial V}{\partial y} + \mathbf{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \mathbf{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \mathbf{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r \frac{\partial V}{\partial r} + \mathbf{a}_\phi \frac{\partial V}{r \partial \phi} + \mathbf{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r}(r A_r) + \frac{\partial A_\phi}{r \partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r} \begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix} = \mathbf{a}_r \left(\frac{\partial A_z}{r \partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \mathbf{a}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \mathbf{a}_z \frac{1}{r} \left[\frac{\partial}{\partial r}(r A_\phi) - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R \frac{\partial V}{\partial R} + \mathbf{a}_\theta \frac{\partial V}{R \partial \theta} + \mathbf{a}_\phi \frac{1}{R \sin\theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2} \frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R \sin\theta} \frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2 \sin\theta} \begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R \sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & R A_\theta & (R \sin\theta) A_\phi \end{vmatrix} = \mathbf{a}_R \frac{1}{R \sin\theta} \left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi} \right]$$
$$+ \mathbf{a}_\theta \frac{1}{R} \left[\frac{1}{\sin\theta} \frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(R A_\phi) \right]$$
$$+ \mathbf{a}_\phi \frac{1}{R} \left[\frac{\partial}{\partial R}(R A_\theta) - \frac{\partial A_R}{\partial \theta} \right]$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial V}{\partial R} \right) + \frac{1}{R^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{R^2 \sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$$

1. This problem is a variant of an EP440 “classic”:

a. Derive the integral form for the following familiar equation of electrostatics, while explaining each step in the process (with just a few words).

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o}$$

b. Name this equation (correctly), and describe its physical meaning.

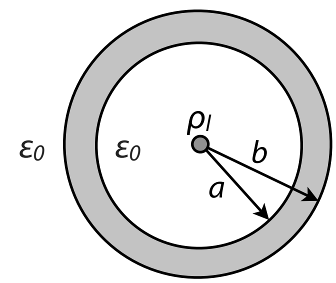
c. Derive the integral form for the following familiar equation of electrostatics, while explaining each step in the process (with just a few words).

$$\nabla \times \mathbf{E} = 0$$

d. Name this equation (correctly), and describe its physical meaning.

2. An infinitely-long *line charge* ρ_l is placed at the center of a *cylindrical* conducting shell, which has inner radius a and outer radius b. See diagram below:

a. Find **E** everywhere (for $R<a$, $a<R<b$, and $R>b$).
(*Note: Box each result!*)

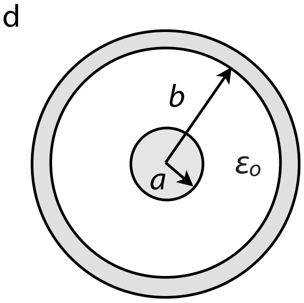


b. Find the induced surface charge density ρ_s at a and b.

c. Explain physically what would happen if you connected the outside of the cylinder to "ground", i.e., a zero potential able to source or sink an unlimited amount of charge.

3. You would like the assess the properties of a *spherical* capacitor , with an inner conductor of radius *a*, a thin outer conducting shell of inside radius *b*, and free space (ε_o) filling the space between. See diagram below:

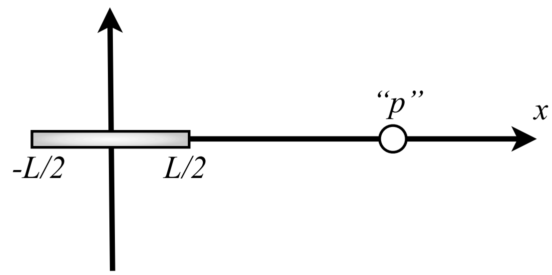
a. Find the capacitance *C* of the system by integrating the electric field to obtain potential.



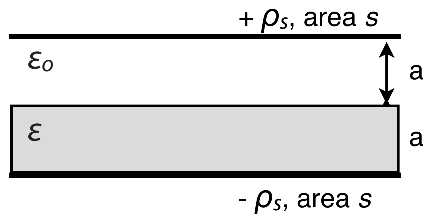
b. Find the capacitance *C* of the system by integrating the electric field *energy density*.

Hints: $w_e = \frac{1}{2}\epsilon_o E^2$ and $W_e = \frac{1}{2}CV^2 = \frac{1}{2}QV$

4. For the charge distribution of length L, with total charge Q, find both the *electric potential* V and the *electric field* **E** at point "p" by any methods that you prefer.



5. Consider the following capacitor, consisting of two parallel plates, a region of free space ε_o, and a region of dielectric ε:



a. In terms of free surface charge ρ_s, write expressions for the **D**, **E**, and **P** fields within the top and bottom regions (i.e., 6 expressions total).

b. The capacitor is charged, and both electrodes are fully isolated from the surroundings to prevent charge transfer. Some kind of wizard magically removes the dielectric material (leaving free space) *without* affecting the total charge on the conductors. Find expressions for the potential difference V before and after.