EP440: ENGINEERING ELECTROMAGNETICS, EXAM #2

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Solve Problem 1, then choose 3 others. Please put a box around all final answers! If attempting more than 4, write "Do Not Grade" on the problem that you do not want to have graded. (Sorry, there is no "best of" option.)

For each problem, show as much work as possible on the pages provided (and the backs of the pages if necessary). You may find the following tables useful.

$$\begin{aligned}
k_e &= \frac{1}{4\pi\epsilon_o} \simeq 9\times 10^9 \quad [\mathrm{N\cdot m^2/C^2}] \\
\epsilon_o &= 8.854\times 10^{-12} \quad [\mathrm{C^2/N\cdot m^2}] \text{ or } [\mathrm{F/m}] \\
\mu_o &= 4\pi\times 10^{-7} \quad [\mathrm{N/A^2}] \text{ or } [\mathrm{H/m}] \\
c &= 2.99792458\times 10^8 \simeq 3\times 10^8 \quad [\mathrm{m/s}] \\
m_e &= 9.109\times 10^{-31} \quad [\mathrm{kg}] \\
m_p &= 1.673\times 10^{-27} \quad [\mathrm{kg}] \\
q_e &= 1.602\times 10^{-19} \quad [\mathrm{C}]
\end{aligned}$$

$$\begin{aligned}
\nabla\times\mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla\times\mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\nabla\cdot\mathbf{D} &= \rho \\
\nabla\cdot\mathbf{B} &= 0
\end{aligned}$$

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B} + \mathbf{E})$$

$$\mathbf{J} = \sigma\mathbf{E},\ \mathbf{D} = \epsilon\mathbf{E},\ \mathbf{B} = \mu\mathbf{H}$$

$$\mathbf{D} = \epsilon_o\mathbf{E} + \mathbf{P} \qquad \mathbf{B} = \mu_o(\mathbf{H} + \mathbf{M})$$

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot\mathbf{J}$$

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$$\begin{aligned}
\int \frac{dx}{x} &= ln x \\
\int e^{ax} &= \frac{1}{a}e^{ax} \\
\int \frac{dx}{\sqrt{a^2 - x^2}} &= \arcsin\frac{x}{a} \\
\int \frac{dx}{\sqrt{x^2 + a^2}} &= \ln\left(x + \sqrt{x^2 + a^2}\right) \\
\int \frac{x\,dx}{\sqrt{x^2 + a^2}} &= \sqrt{x^2 + a^2} \\
\int \frac{dx}{x^2 + a^2} &= \frac{1}{a}\arctan\frac{x}{a} \\
\int \frac{dx}{(x^2 + a^2)^{3/2}} &= \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}} \\
\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} &= -\frac{1}{\sqrt{x^2 + a^2}}
\end{aligned}$$

$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B}$

$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

$\nabla(\psi V) = \psi\nabla V + V\nabla\psi$

$\nabla\cdot(\psi\mathbf{A}) = \psi\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla\psi$

$\nabla\times(\psi\mathbf{A}) = \psi\nabla\times\mathbf{A} + \nabla\psi\times\mathbf{A}$

$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

$\nabla\cdot\nabla V = \nabla^2 V$

$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

$\nabla\times\nabla V = 0$

$\nabla\cdot(\nabla\times\mathbf{A}) = 0$

$\int_v \nabla\cdot\mathbf{A}\,dv = \oint_s \mathbf{A}\cdot d\mathbf{s}$ (Divergence theorem)

$\int_s \nabla\times\mathbf{A}\cdot d\mathbf{s} = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell}$ (Stokes's theorem)

$$E_{1t} = E_{2t} \qquad \mathbf{a}_{n2}\cdot(\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$$

$$B_{1n} = B_{2n} \qquad \mathbf{a}_{n2}\times(\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

$$U_L = \frac{1}{2}LI^2 \qquad U_C = \frac{1}{2}CV^2$$

$$w_m = \frac{1}{2}\mathbf{H}\cdot\mathbf{B} = \frac{B^2}{2\mu} = \frac{1}{2}\mu H^2$$

$$w_e = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} = \frac{D^2}{2\epsilon} = \frac{1}{2}\epsilon E^2$$

$$-\oint_S (\mathbf{E}\times\mathbf{H})\cdot d\mathbf{s} = \frac{\partial}{\partial t}\int_V (w_e + w_m)dv + \int_V \sigma E^2 dv$$

Cartesian Coordinates (x, y, z)

$$\nabla V = \mathbf{a}_x\frac{\partial V}{\partial x} + \mathbf{a}_y\frac{\partial V}{\partial y} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r\frac{\partial V}{\partial r} + \mathbf{a}_\phi\frac{\partial V}{r\partial\phi} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \mathbf{a}_r\left(\frac{\partial A_z}{r\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{a}_z\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R\frac{\partial V}{\partial R} + \mathbf{a}_\theta\frac{\partial V}{R\partial\theta} + \mathbf{a}_\phi\frac{1}{R\sin\theta}\frac{\partial V}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix} = \mathbf{a}_R\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]$$
$$+ \mathbf{a}_\theta\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\phi} - \frac{\partial}{\partial R}(RA_\phi)\right]$$
$$+ \mathbf{a}_\phi\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

1. This problem is another variant of an EP440 "classic".

a. Derive the familiar integral form from the following "point form" expression of Ampere's law, while explaining each step in the process (with just a few words).

$$\nabla \times \mathbf{B} = \mu_o \mathbf{J}$$

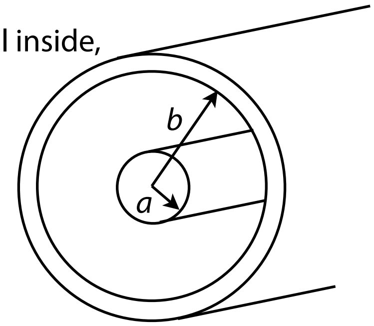
b. Derive a "point form" expression for Ampere's law in terms of magnetic field intensity **H**:

c. Show that magnetic flux can be expressed in terms of magnetic vector potential **A** (Recall that: $\mathbf{B} = \nabla \times \mathbf{A}$):

$$\Phi = \oint \mathbf{A} \cdot d\mathbf{l}$$

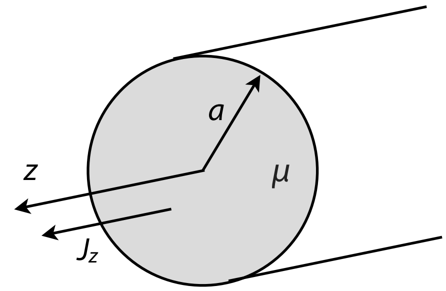
2. Assess the electrical properties of a coaxial cable (made up of coaxial cylinders) of very long length, with an inner conductor of radius a, a thin outer conducting shell of inside radius b, and free space in-between. See diagram below:

a. Solve the cylindrical *Laplace equation* to obtain the potential inside, given the boundary conditions that $V(a)=V_o$ and $V(b)=0$.



b. *Using your result above*, find the electric field inside the coaxial cable.

3. A long cylinder, with radius a, is aligned along the z-axis and constructed from a conducting magnetic material with permeability μ. It carries *uniform* current density $\mathbf{J}=\hat{a}_z J_z$ and thus total current $I=J_z\pi a^2$.



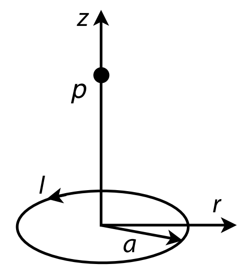
a. What are the **H, B**, and **M** fields inside the cylinder?

b. What is the magnetization current density $\mathbf{J_m}$ inside the cylinder (Hint: Not zero!)?

c. What is the magnetization surface current density $\mathbf{J_{ms}}$ at $r=a$?

4. Refer to the diagram to the right (point "*p*" is at height *z* above center of loop with radius *a*):

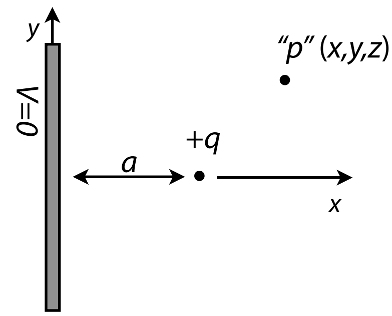
a. Find the vector potential **A** at point "*p*" due to current *I*.



b. Does the vector potential **A** satisfy the Coulomb gauge condition of non-divergence?

c. Find the magnetic flux density **B** at point *p* using *any method that you prefer.*

5. A positive charge q is placed near an infinite ground plane.



a. Find the scalar electric potential at any point "p".

b. Find the vector electric field *at the origin*.

c. Find the *peak* surface charge density ρ_s on the infinite ground plane boundary.