Problem 1

P.8-19 From Gauss's law, $\bar{E} = \bar{a}_r \frac{\rho}{2\pi\epsilon r}$, where ρ is the line charge density on the inner conductor.

$$V_0 = -\int_b^a \bar{E} \cdot d\bar{r} = \frac{\rho}{2\pi\epsilon} \ln\left(\frac{b}{a}\right) \longrightarrow \bar{E} = \bar{a}_r \frac{V_0}{r \ln(b/a)}.$$

From Ampère's circuital law, $\bar{H} = \bar{a}_\phi \frac{I}{2\pi r}$.

Poynting vector, $\bar{\mathcal{P}} = \bar{E} \times \bar{H} = \bar{a}_z \frac{V_0 I}{2\pi r^2 \ln(b/a)}$.

Power transmitted over cross-sectional area:

$$P = \int_S \bar{\mathcal{P}} \cdot d\bar{s} = \frac{V_0 I}{2\pi \ln(b/a)} \int_0^{2\pi} \int_a^b \left(\frac{1}{r^2}\right) r\, dr\, d\phi = V_0 I.$$

Figure 1: Cheng Solution for P.8-19

Problem 2

P.8-20 a) $\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$, $f = 10^4/2\pi$.

b) $\bar{H}(z,t) = \bar{a}_y H_0 e^{-z/\delta} \cos\left(10^4 t - \frac{z}{\delta}\right)$,

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{1}{\sigma\delta} = \frac{\sqrt{2}}{\sigma\delta} e^{j\pi/4}.$$

$$\bar{E}(z,t) = \bar{a}_x \frac{\sqrt{2}}{\sigma\delta} H_0 e^{-z/\delta} \cos\left(10^4 t - \frac{z}{\delta} + \frac{\pi}{4}\right).$$

c) $$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e\,(\bar{E} \times \bar{H}^*) = \bar{a}_z \frac{1}{2} \frac{\sqrt{2}}{\sigma\delta} H_0^2 \cos\frac{\pi}{4}$$
$$= \bar{a}_z \frac{1}{2}\left(\frac{H_0^2}{\sigma\delta}\right) \quad (\mathrm{W/m^2}).$$

Figure 2: Cheng Solution for P.8-20

Problem 3

P.8-25 a) From Eqs. (8-128) and (8-129):

$$\bar{E}_1(x,z;t) = -2E_{i0}\Big[\bar{a}_x \cos\theta_i \sin(\beta_1 z\cos\theta_i)\cos(\omega t-\beta_1 x\sin\theta_i)$$
$$+\ \bar{a}_z \sin\theta_i \cos(\beta_1 z\cos\theta_i)\sin(\omega t-\beta_1 x\sin\theta_i)\Big],$$

$$\bar{H}_1(x,z;t) = \bar{a}_y \frac{2E_{i0}}{\eta_1}\cos(\beta_1 z\cos\theta_i)\sin(\omega t-\beta_1 x\sin\theta_i).$$

b) $$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_x \frac{2E_{i0}^2}{\eta_1}\sin\theta_i\cos^2(\beta_1 z\cos\theta_i).$$

Figure 3: Cheng Solution for P.8-25

Problem 4

P.8-28 a) $$\Gamma = \frac{E_r}{E_i} = \frac{\eta_c-\eta_0}{\eta_c+\eta_0}\ ; \qquad \eta_0=\sqrt{\frac{\mu_0}{\epsilon_0}}=120\pi,\ \eta_c=\sqrt{j\omega\mu/\sigma}.$$
$$|\eta_c| << \eta_0.$$

b) $$|\Gamma|^2 = \left|\frac{\eta_c-\eta_0}{\eta_c+\eta_0}\right|^2 = \left|\frac{1-\eta_c/\eta_0}{1+\eta_c/\eta_0}\right|^2 \cong \left|1-2\eta_c/\eta_0\right|^2$$
$$=(1-2\eta_c/\eta_0)(1-2\eta_c^*/\eta_0) \cong 1-4\mathcal{R}e(\eta_c)/\eta_0.$$

Fraction of power absorbed, $F = 1-|\Gamma|^2 = \frac{4}{\eta_0}\mathcal{R}e\sqrt{\frac{j\omega\mu}{\sigma}}$
$$= \frac{4}{\eta_0}\sqrt{\frac{\omega\mu}{2\sigma}}.$$

c) $\omega = 2\pi\times 10^6$ (Hz). For iron: $\mu = 4000\times(4\pi 10^{-7})$ (H/m), $\sigma = 10^7$ (S/m).

$F = 4.21\times 10^{-4}$, or 0.0421%.

Figure 4: Cheng Solution for P.8-28

Problem 5

(Byron's solution follows...) All of these parts consist of ensuring that the character impendence Z_0 of the stripline is constant. In this case we approximate the stripline as a parallel plate transmission line, which is not necessarily great but its all we can do without computer models.

Part a

Doubling the dielectric constant and changing the distance between the plates gives:

$$Z_0' = Z_0$$

$$\frac{d'}{w}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}$$

Canceling like terms and rearranging:

$$\boxed{d' = \sqrt{2}d}$$

part b

Doubling the dielectric constant and changing the width of the plates gives:

$$Z_0' = Z_0$$

$$\frac{d}{w'}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}$$

Canceling like terms and rearranging:

$$\boxed{w' = \frac{1}{\sqrt{2}}d}$$

part c

Doubling the dielectric constant and changing the width of the plates gives:

$$Z_0' = Z_0$$

$$\frac{d}{w'}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}$$

Canceling like terms and rearranging:

$$\boxed{w' = \frac{1}{\sqrt{2}}w}$$

part d

The velocity of propagation v_p in a lossless transmission line is only dependent on the inductance and capacitance of the transmission line, of which it follows that it only depends on the dielectric and diamagnetic constants of the media.

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}$$

Since the dielectric constant only changes in parts a and b of the problem, the speed of propagation only changes in parts a and b. If we define the normal speed of propagation of our unmodified stripline to be $v_{p,0}$, then mathematically:

$$\boxed{v_{p,a} = \frac{1}{\sqrt{2}} v_{p,0}} \qquad \boxed{v_{p,b} = \frac{1}{\sqrt{2}} v_{p,0}} \qquad \boxed{v_{p,c} = v_{p,0}}$$

P.9-3 Eq. (9-20): $Z_0 = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}$.

a) $Z_0 = \frac{d'}{w}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}. \longrightarrow d' = \sqrt{2}\, d.$

b) $Z_0 = \frac{d}{w'}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}. \longrightarrow w' = \frac{1}{\sqrt{2}} w.$

c) $Z_0 = \frac{2d}{w'}\sqrt{\frac{\mu}{\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow w' = 2w.$

d) $u_p = \frac{1}{\sqrt{\mu\epsilon}} \longrightarrow$ $u_{pa} = u_p/\sqrt{2}$ for case a.
$u_{pb} = u_p/\sqrt{2}$ for case b.
$u_{pc} = u_p$ for case c.

Figure 5: Cheng Solution for P.9-3

Problem 6

P.9-4 Given: $\sigma_c = 1.6 \times 10^7$ (S/m), $w = 0.02$ (m), $d = 2.5 \times 10^{-3}$ (m).
Lossy dielectric slab: $\mu = \mu_0$, $\epsilon_r = 3$, $\sigma = 10^{-3}$ (S/m).
$f = 5 \times 10^8$ (Hz).

a) $R = \frac{2}{w}\sqrt{\frac{\pi f \mu_0}{\sigma_c}} = 1.11$ (Ω/m)

$L = \mu \frac{d}{w} = 0.157$ (μH/m)

$G = \sigma \frac{w}{d} = 0.008$ (S/m)

$C = \epsilon \frac{w}{d} = 0.212$ (nF/m).

b) $\frac{|E_z|}{|E_y|} = \sqrt{\frac{\omega\epsilon}{\sigma_c}} = 4.167 \times 10^{-5}$.

c) $\omega L = 493.5 \gg R$, $\omega C = 0.667 \gg G$

$\gamma \cong j\omega\sqrt{LC}\left[1 + \frac{1}{2j}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right] = 0.129 + j18.14$ (m^{-1})

$Z_0 \cong \sqrt{\frac{L}{C}}\left[1 + \frac{1}{2j}\left(\frac{R}{\omega L} - \frac{G}{\omega C}\right)\right] = 27.21 + j0.13$ (Ω).

Figure 6: Cheng Solution for P.9-4

Problem 7

P.9-10 a) For two-wire transmission line:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{\pi}\sqrt{\frac{\mu}{\epsilon}}\cosh^{-1}\left(\frac{D}{2a}\right) = \frac{120}{\sqrt{\epsilon_r}}\ln\left[\frac{D}{2a} + \sqrt{\left(\frac{D}{2a}\right)^2 - 1}\right] = 300 \ (\Omega).$$

$\frac{D}{2a} = 21.27 \longrightarrow D = 25.5 \times 10^{-3}$ (m).

b) For coaxial transmission line:

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\left(\frac{b}{a}\right) = \frac{40}{\sqrt{\epsilon_r}}\ln\left(\frac{b}{a}\right) = 75.$$

$\frac{b}{a} = 6.52 \longrightarrow b = 3.91 \times 10^{-3}$ (m).

Figure 7: Cheng Solution for P.9-10