Problem 1

P.5-3 R_1 = Resistance per unit length of core $= \frac{1}{\sigma S_1} = \frac{1}{\sigma \pi a^2}$.

R_2 = Resistance per unit length of coating $= \frac{1}{0.1\sigma S_2}$.

Let b = Thickness of coating. $\longrightarrow S_2 = \pi(a+b)^2 - \pi a^2 = \pi(2ab+b^2)$.

a) $R_1 = R_2 \longrightarrow b = (\sqrt{11} - 1)a = 2.32a$.

b) $I_1 = I_2 = \frac{I}{2}$. $\quad J_1 = \frac{I}{2\pi a^2}$, $\quad J_2 = \frac{I}{2S_2} = \frac{I}{20S_1} = \frac{I}{20\pi a^2}$.

$E_1 = \frac{J_1}{\sigma} = \frac{I}{2\pi a^2 \sigma}$, $E_2 = \frac{J_2}{0.1\sigma} = \frac{I}{2\pi a^2 \sigma}$.

Thus, $J_1 = 10 J_2$ and $E_1 = E_2$.

Problem 2

For this problem we have 3 different regions; it best to be approached as changing the current enclosed for each region, and using the following relation derived from Ampere's law:

$$B_\phi = \frac{\mu_o I_{encl}}{2\pi r} \tag{1}$$

So, for the first region (r¡a), we end up with the following enclosed current:

$$I_{encl} = \frac{r^2}{a^2} I \tag{2}$$

Which results in the following equation for the B-field:

$$B_\phi = \frac{\mu_o I r}{2\pi a^2} \tag{3}$$

For the following region (a¡r¡b), the enclosed current is equal to I, so its just an easy substitution. The last region is bit tricky and its enclosed current is the following:

$$I_{encl} = I\left(1 - \frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)}\right) \tag{4}$$

which when subbed into the equation gives the following:

$$B_\phi = \frac{\mu_o I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right) \tag{5}$$

Problem 3

This problem involves a lot of Biot-Savart in different scenarios. The general form of Biot-Savart is as follows:

$$d\mathbf{B} = \frac{\mu_o I}{4\pi}\left(\frac{dl' \times \mathbf{R}}{R^3}\right) \tag{6}$$

a) Let L → a, L → b and use superposition

$$\bar{B} = \frac{2\mu_o I}{2\pi}\left\{\frac{a/2}{[(\frac{a}{2})^2+(\frac{b}{2})^2]^{1/2}\ \frac{b}{2}} + \frac{b/2}{[(\frac{a}{2})^2+(\frac{b}{2})^2]^{1/2}\ \frac{a}{2}}\right\}\bar{i}_z$$

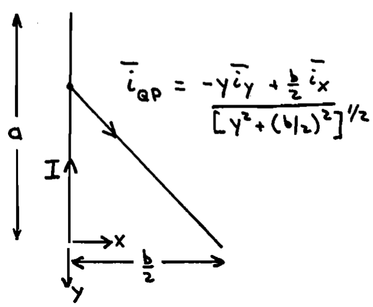
$$= \frac{2\mu_o I}{\pi[a^2 + b^2]^{1/2}}\left[\frac{a}{b} + \frac{b}{a}\right]\bar{i}_z = \frac{2\mu_o I[a^2 + b^2]^{1/2}}{\pi ab}\bar{i}_z$$

e) No contribution from semi-infinite line currents.

From horizontal length

$$B_z = \frac{\mu_o I \frac{b}{2}}{2\pi a[(\frac{b}{2})^2 + a^2]^{1/2}}$$

From vertical length



$$\bar{B} = \frac{\mu_o I}{4\pi}\int_{-a}^{0}\frac{-\bar{i}_y \times \bar{i}_{QP}\,dy}{[y^2 + (\frac{b}{2})^2]}$$

$$= \frac{\mu_o I\bar{i}_z}{4\pi}\int_{-a}^{0}\frac{\frac{b}{2}\,dy}{[y^2 + (\frac{b}{2})^2]^{3/2}}$$

$$= \frac{\mu_o I}{4\pi}\bar{i}_z\,\frac{b}{2}\,\frac{y}{(\frac{b}{2})^2[y^2+(\frac{b}{2})^2]^{1/2}}\Bigg|_{y=-a}^{0}$$

$$= \frac{\mu_o I\, a\,\bar{i}_z}{2\pi b[a^2 + (\frac{b}{2})^2]^{1/2}}$$

$$B_{z\text{total}} = \frac{\mu_o I}{2\pi\left[a^2 + (\frac{b}{2})^2\right]^{1/2}}\left[\frac{2a}{b} + \frac{b}{2a}\right] = \frac{\mu_o I}{\pi ab}\left[a^2 + (\frac{b}{2})^2\right]^{1/2}$$

Now for the last one we can approximate the contribution from the 2 line segments through Ampere's law and treating them as infinitely long. The portion from the arc can be dealt with using Biot-Savart and an integral from $0-\pi$, which results in the following equation:

$$\mathbf{B} = \frac{\mu_o I}{4\pi}\hat{a}_z \tag{7}$$

Now, putting them together:

$$\mathbf{B} = \frac{\mu_o I}{2a}\left(\frac{1}{2} + \frac{1}{\pi}\right)\hat{a}_z \tag{8}$$

Problem 4

19. a) $$A_z = \frac{\mu_o I}{4\pi}\left\{\sinh^{-1}\frac{\frac{L}{2} - z}{r} + \sinh^{-1}\frac{\frac{L}{2} + z}{r}\right\}$$

$$\nabla\cdot\overline{A} = \frac{\partial A_z}{\partial z} = \frac{\mu_o I}{4\pi}\left\{\frac{-1}{\sqrt{r^2 + (\frac{L}{2} - z)^2}} + \frac{1}{\sqrt{r^2 + (\frac{L}{2} + z)^2}}\right\} \neq 0$$

Problem 5

P.6-14 $B_\phi = \frac{\mu_0 NI}{2\pi r}$, $\quad \Phi = \int_S B_\phi\, ds = \frac{\mu_0 NI}{2\pi}\int_a^b \frac{h}{r}\,dr = \frac{\mu_0 NIh}{2\pi}\ln\frac{b}{a}$.

If B_ϕ at $r = \frac{a+b}{2}$ is used, $\Phi' = \frac{\mu_0 NIh}{\pi}\left(\frac{b-a}{b+a}\right)$.

$$\%\ error = \frac{\Phi' - \Phi}{\Phi} \times 100\% = \left[\frac{2(b-a)}{(b+a)\ln(b/a)} - 1\right] \times 100\%.$$