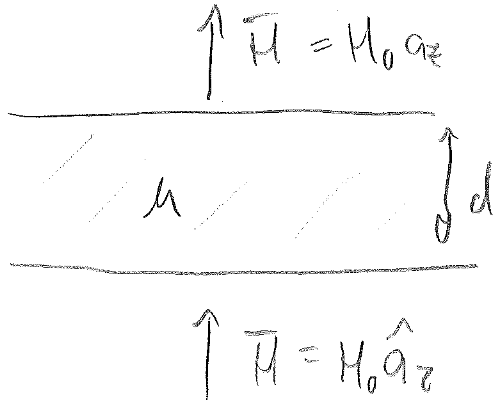
Magnetic Material Slab: (Cheng P.6-21)

Cheng P.6.21

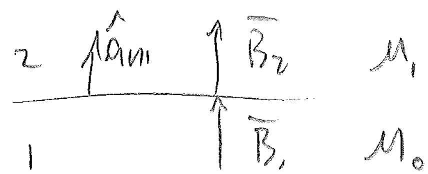


What is $\bar{B}$ in the slab? (μ)

$\bar{B} = \mu \bar{H}$

for normal B-field,

$\hat{a}_{n_1} \cdot (\bar{B}_2 - \bar{B}_1) = 0$

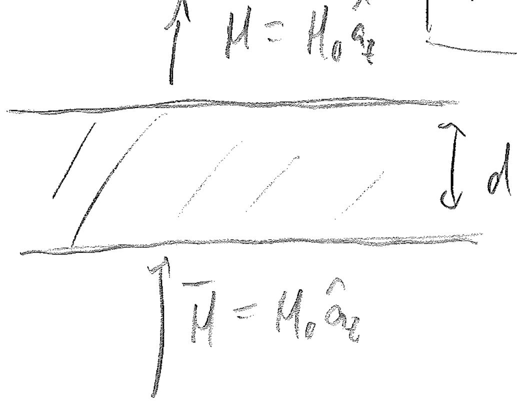


$$\hat{a}_z \cdot (B_2 \hat{a}_z - B_1 \hat{a}_z) = 0$$

$$\hat{a}_z \cdot (H \mu \hat{a}_z - H_0 \mu_0 \hat{a}_z) = 0$$

$$H \mu = H_0 \mu_0$$

$$\boxed{H = \frac{\mu_0}{\mu} H_0}$$



$\bar{M}_i = a_z M_i$

$$\hat{a}_z \cdot (\mu_0 (H + M_i) \hat{a}_z - \mu_0 H_0) = 0$$

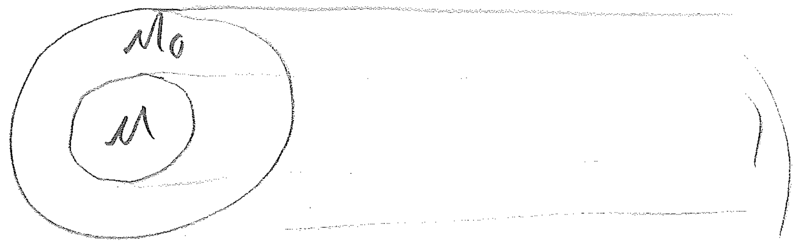
$$H + M_i = H_0$$

$$\boxed{\bar{H} = \hat{a}_z (H_0 - M_i)}$$

for $H_0 = 0$, $\bar{H}$ in slab $= -\bar{M}_i$!

$\therefore \bar{B} = 0$ inside

(5)
Cheng P. 6-22 Solenoid ✓



for $\underline{r < a}$:
$$\bar{H} = \hat{a}_z n I$$
$$\bar{B} = \mu \bar{H} = \hat{a}_z \mu n I$$
$$\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \left(\frac{\mu}{\mu_0} n I - n I\right)\hat{a}_z$$
$$\bar{M} = \left(\frac{\mu}{\mu_0} - 1\right) n I \hat{a}_z$$

for $\underline{a < r < b}$:
$$\bar{H} = \hat{a}_z n I$$
$$\bar{B} = \hat{a}_z \mu_0 n I$$
$$\bar{M} = 0 \text{ (because free space)}$$

In material μ,

$$\bar{J}_m = \nabla \times \bar{M} \rightarrow \bar{M} = \text{constant},$$
thus curl-free $\boxed{\bar{J}_m = 0}$

$$\bar{J}_{ms} = \bar{M} \times \hat{a}_n = \left(\frac{\mu}{\mu_0} - 1\right) I n \underbrace{\hat{a}_z \times \hat{a}_r}$$

$$\boxed{\bar{J}_{ms} = \left(\frac{\mu}{\mu_0} - 1\right) I n \, \hat{a}_\phi}$$

Zahn 5.24

24. a) $\oint_L \overline{H}\cdot\overline{d\ell} = \int_S \overline{J}_f\cdot\overline{dS} \rightarrow H_\phi(2\pi r) = \begin{cases} J_o\pi r^2 & r < a \\ J_o\pi a^2 & r > a \end{cases}$

$$H_\phi = \begin{cases} \dfrac{J_o r}{2} \\ \dfrac{J_o a^2}{2r} \end{cases} ; \quad B_\phi = \begin{cases} \dfrac{\mu J_o r}{2} \\ \dfrac{\mu_o J_o a^2}{2r} \end{cases} ; \quad M_\phi = \frac{B_\phi}{\mu_o} - H_\phi = \begin{cases} \dfrac{(\mu - \mu_o)}{\mu_o}\dfrac{J_o r}{2} & r < a \\ 0 & r > a \end{cases}$$

$$\overline{J}_m = \nabla \times \overline{M} = \frac{\mu - \mu_o}{\mu_o} J_o \overline{i}_z$$

$$K_{mz} = -M_\phi(r=a_-) = -\left(\frac{\mu - \mu_o}{\mu_o}\right)\frac{J_o a}{2}$$

b) $H_x = \begin{cases} -\dfrac{K_o}{2} & y > 0 \\ +\dfrac{K_o}{2} & y < 0 \end{cases} ; \quad B_x = \begin{cases} -\dfrac{\mu K_o}{2} & 0 < y < \dfrac{d}{2} \\ \dfrac{\mu K_o}{2} & -\dfrac{d}{2} < y < 0 \\ -\dfrac{\mu_o K_o}{2} & y > \dfrac{d}{2} \\ \dfrac{\mu_o K_o}{2} & y < -\dfrac{d}{2} \end{cases} ; \quad M_x = \begin{cases} -\dfrac{(\mu-\mu_o)K_o}{2\mu_o} & 0 < y < \dfrac{d}{2} \\ \dfrac{(\mu-\mu_o)K_o}{2\mu_o} & -\dfrac{d}{2} < y < 0 \\ 0 & |y| > \dfrac{d}{2} \end{cases}$

$$K_{zm}(y=0) = M_x(y=0_+) - M_x(y=0_-) = -\frac{(\mu - \mu_o)K_o}{\mu_o}$$

$$K_{zm}\left(y=\frac{d}{2}\right) = M_x\left(y=\frac{d_+}{2}\right) - M_x\left(y=\frac{d_-}{2}\right) = \frac{\mu - \mu_o}{2\mu_o}K_o$$

$$K_{zm}\left(y=-\frac{d}{2}\right) = M_x\left(y=-\frac{d_+}{2}\right) - M_x\left(y=-\frac{d_-}{2}\right) = \frac{\mu - \mu_o}{2\mu_o}K_o$$

Cheng 6.1 / Zahn 5.1.

Section 5.1

1. a) $v_x = v_{xo}\cos\omega_o t + v_{yo}\sin\omega_o t$; $\omega_o = qB_o/m$

$v_y = v_{yo}\cos\omega_o t - v_{xo}\sin\omega_o t$

$v_z = v_{zo}$

$$x = \int v_x dt = x_o + \frac{v_{yo}}{\omega_o} + \frac{1}{\omega_o}[-v_{yo}\cos\omega_o t + v_{xo}\sin\omega_o t]$$

$$y = \int v_y dt = y_o - \frac{v_{xo}}{\omega_o} + \frac{1}{\omega_o}[v_{yo}\sin\omega_o t + v_{xo}\cos\omega_o t]$$

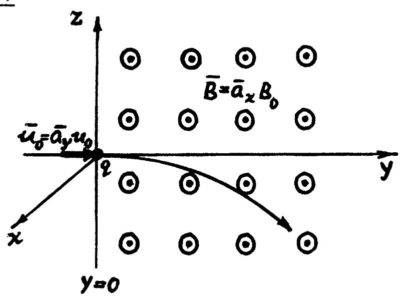
$$z = \int v_z dt = v_{zo}t + z_o$$

b) $$[x - (x_o + \frac{v_{yo}}{\omega_o})]^2 + [y - (y_o - \frac{v_{xo}}{\omega_o})]^2 = \frac{v_{xo}^2 + v_{yo}^2}{\omega_o^2}$$

Equation of circle with radius $r = [(v_{xo}^2 + v_{yo}^2)/\omega_o^2]^{1/2}$ with center at $(x_o + \frac{v_{yo}}{\omega_o},\ y_o - \frac{v_{xo}}{\omega_o})$.

c) $$\frac{1}{2} m|\bar{v}|^2 = \frac{1}{2} m[v_{xo}^2 + v_{yo}^2 + v_{zo}^2]$$

P. 6-1



$$\frac{du_y}{dt} = \frac{qB_0}{m} u_z = \omega_0 u_z, \quad ①$$

$$\frac{du_z}{dt} = -\frac{qB_0}{m} u_y = -\omega_0 u_y, \quad ②$$

$$\omega_0 = qB_0/m.$$

Combining ① and ②:

$$\frac{d^2 u_z}{dt^2} + \omega_0^2 u_z = 0$$

$$\longrightarrow u_z = A\cos\omega_0 t + B\sin\omega_0 t.$$

At $t=0$, $u_z=0 \longrightarrow A=0$; $u_z = B\sin\omega_0 t$.

Substituting u_z in ②: $u_y = -B\cos\omega_0 t$. At $t=0$, $u_y = u_0 \longrightarrow B = -u_0$.

$$\therefore\ u_y = u_0\cos\omega_0 t \longrightarrow y = \frac{u_0}{\omega_0}\sin\omega_0 t, \quad (t=0,\ y=0); \quad ③$$

$$u_z = -u_0\sin\omega_0 t \longrightarrow z = \frac{u_0}{\omega_0}\cos\omega_0 t + c_1 \quad \left(t=0,\ z=0 \rightarrow c_1 = -\frac{u_0}{\omega_0}\right).$$

$$= -\frac{u_0}{\omega_0}(1-\cos\omega_0 t). \quad ④$$

From ③ and ④: $y^2 + \left(z - \frac{u_0}{\omega_0}\right)^2 = \left(\frac{u_0}{\omega_0}\right)^2$ --- Eq. of a shifted circle.